

1. Recap.

From last class we learned:

- We defined the area accumulation function $F(x) = \int_a^x f(t) dt$, which accumulates the area under f from a (a constant) to x (a variable).
- (FTC, part 1) We showed that the rate at which the area is cumulating is the function whose area is being accumulated. In other words, $F'(x) = f(x)$.
- This shows that the area accumulation function (the integral) is an antiderivative of f .

2. Fundamental Theorem of Calculus, Part 2.

A corollary of FTC (another version of FTC) from last class allows us to use antiderivatives to calculate integrals:

Theorem (FTC, Part 2) If $f(x)$ is continuous on $[a, b]$ and $F(x)$ is any antiderivative on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a).$$

How do we justify the second part of the Fundamental Theorem of Calculus using the first part of the theorem?

3. Some computations.

Compute the following definite integrals using the second part of the FTC.

(a) $\int_0^\pi e^x + \cos x - 1 dx$

$$(b) \int_0^{\pi/4} \frac{1}{\cos^2 x} + \pi - x \, dx$$

$$(c) \int_1^4 \sqrt{x} + \frac{1}{x} \, dx$$

$$(d) \int_1^{16} x^3 + \sin x + \frac{1}{x^3} \, dx$$

$$(e) \int_{-4}^4 |x| \, dx$$

$$(f) \int_{-1}^0 \pi^{x-1} \, dx$$