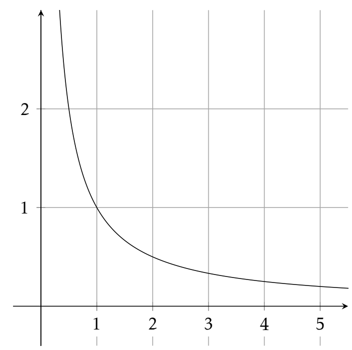
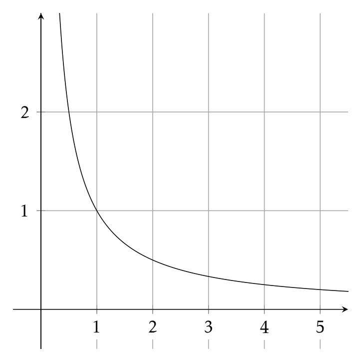
1. Sums for $1/x$.

Let us consider the function $f(x) = \frac{1}{x}$. We want to estimate the area under this function between $x = 2$ and $x = 5$. Our goal is to make the connection between the sums below and various estimations of this area. For each of the following sums explain in words and pictures what they correspond to.

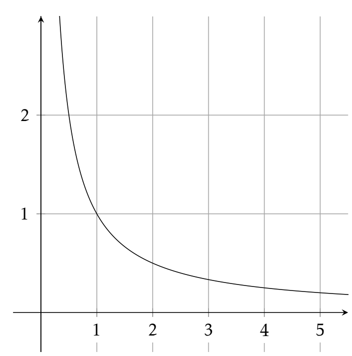
(a) $\sum_{k=0}^{2} f(2+k) \cdot 1$



(b) $\sum_{k=1}^{3} f(2+k) \cdot 1$



(c) $\sum_{k=1}^{3} f\left(2 + k\frac{5-2}{3}\right) \cdot \left(\frac{5-2}{3}\right)$



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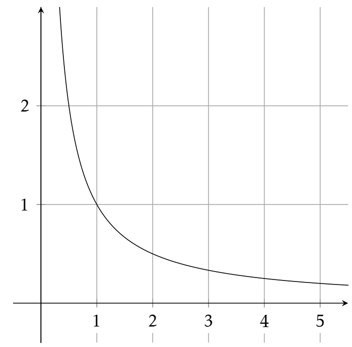
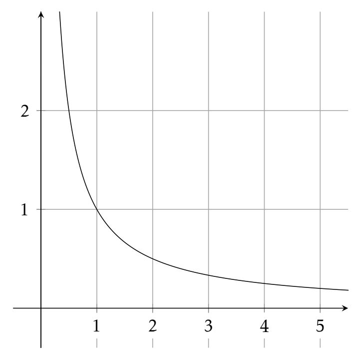
(d) $\sum_{k=1}^{3} f\left(1.5 + k\frac{5-2}{3}\right) \cdot \left(\frac{5-2}{3}\right)$



(e) $\sum_{k=1}^{6} f\left(2 + k\frac{5-2}{6}\right) \cdot \left(\frac{5-2}{6}\right)$



(f) Write down the sum for an approximation of the area under $f(x) = 1/x$ between $x = 2$ and $x = 5$ using 10 right-handed rectangles.

(g) Same question but for 1000 rectangles. Then, how can we write thus sum for “infinitely many” rectangles?

2. Abstract sums.

(a) By inferring from the previous parts, write down a general formula (using $\sum$ notation) for the approximation obtained using k many left-handed rectangles of equal width. Your answer should be in terms of f, a, b and k.

(b) By modifying your previous answer, come up with a general formula for the approximation obtained by using k many right-handed rectangles of equal width. How about k many midpoint rectangles of equal width?

(c) We have discussed the following approximation schemes involving rectangles: left-handed, right-handed, and midpoint. As you can see from the formulas above, these differ based on which point we choose in each interval: the left endpoint, right endpoint, or midpoint.

These schemes can give underestimates or overestimates of the desired area, depending on the shape of f. Can you come up with another approximation scheme involving rectangles which will *always* give an underestimate for the desired area, regardless of the shape of f?