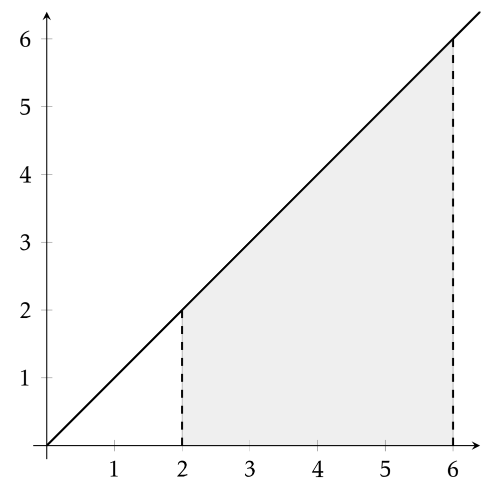
Let us start by estimating the area under a very simple "curve", namely the function $f(x) = x$. We want to approximate the area under this function between $x = 2$ and $x = 6$.
Let us underline that we want to find a **method that works for all kinds of functions and curves**, not only straight lines like this one (where we actually know how to compute the area).

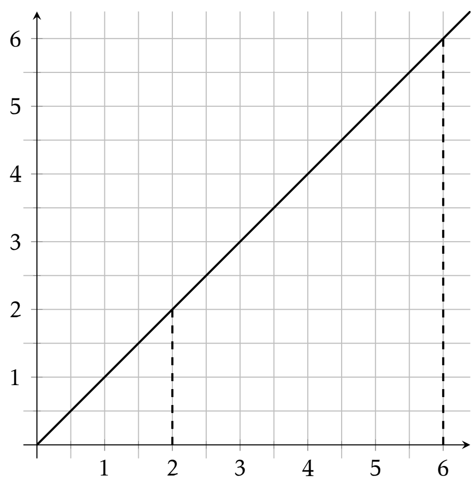


1. Find as many ways as you can to estimate this area. In each case, determine if you are overestimating or under estimating the area, and how you could improve your estimate.

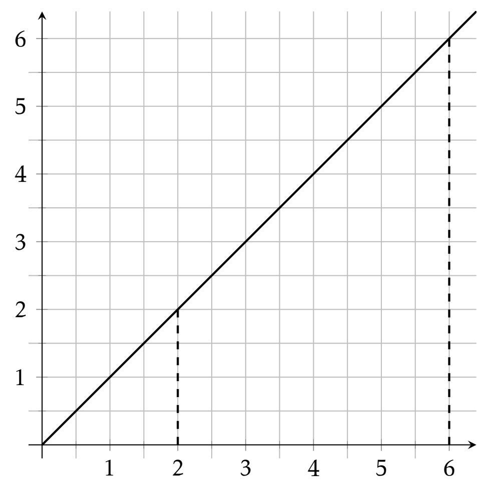
Three important ways of approximating an area under a curve are to use left-handed rectangles, right-handed rectangles, and mid-point rectangles.

2. On the graphs below, estimate the area under the function $f(x) = x$ between $x = 2$ and $x = 6$ using four rectangles with bases in the intervals [2, 3], [3, 4], [4, 5], [5, 6].
   In each case, draw the rectangles and compute the estimated area.

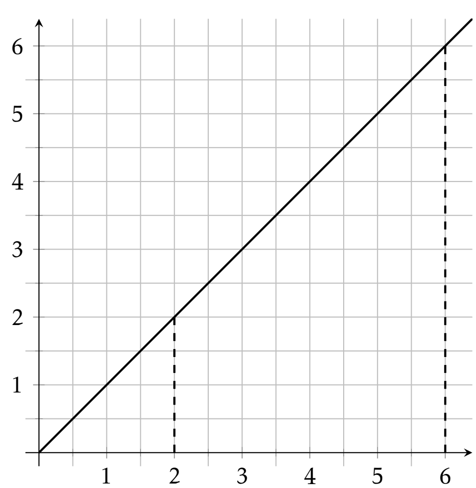
a) Left-handed rectangles



b) Right-handed rectangles

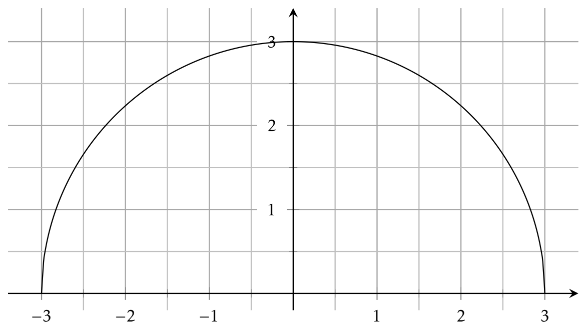


c) Midpoint rectangles

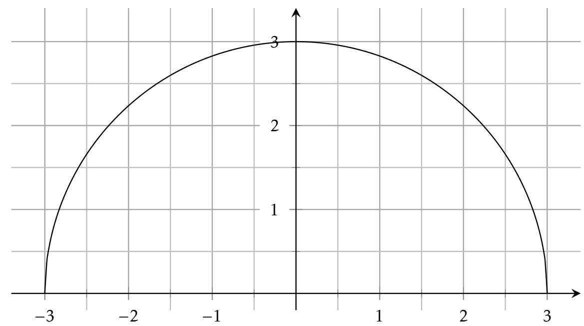


3. Let us now consider the half-circle whose equation is $f(x) = \sqrt{9 - x^2}$.
   We will compare the left-handed and right-handed rectangles. Before doing any computation, which one of these to sums do you think is going to be the biggest?

a) Left-handed rectangles



b) Right-handed rectangles



4. What do you get? How do you explain your result?

5. How could we improve the approximations for these areas? List as many ways as you can.

6. The velocity of a particle is give by the functions $y = \frac{1}{x}$. Approximate the distance it has traveled between time $t = 2$s and $t = 5$s (use 3 rectangles for that).