Math 1110: **Indeterminate Forms (Section 4.5)** Oct 29 (Mon)

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1. Objectives.

At the end of this section, you will be able to:

- explain in words what an indeterminate form is,
- explain what L'Hôpital's rule is, when we can use it, and what kinds of limits we can compute with it,
- correctly use L'Hôpital's rule to compute limits.

2. Indeterminate forms.

(a) If we want to compute $\lim_{x\to\infty} \frac{x}{x^2-9}$, what happens when you try to "plug in" $x = \infty$? What "algebraic manipulation" can we do to compute this limit?

(b)

> Definition: $\lim_{x\to a} \frac{f(x)}{g(x)}$ has indeterminate form $\frac{0}{0}$ if $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$.
> Similarly, we can define indeterminate forms $\frac{\infty}{\infty}$, $\infty \cdot 0$, $\infty - \infty$, 0^0, 1^∞, ∞^0.

For each of the following limits, write down their indeterminate form if applicable.

$$\lim_{x\to 0} \frac{3x - \sin x}{x} \qquad \lim_{x\to\infty} \frac{e^x}{x^2} \qquad \lim_{x\to 1} \frac{\ln x}{x-1} \qquad \lim_{x\to 0} x\sin(1/x)$$

3. L'Hôpital's rule.

> Suppose that $\lim_{x\to a} \frac{f(x)}{g(x)}$ has indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$. Suppose also that there is an open interval I containing a such that:
>
> 1. f and g are differentiable on I;
> 2. g' is never 0 on I (except maybe at a).
>
> Then
> $$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)},$$
> provided the limit on RHS exists.

What do you have to check before applying l'Hôpital's rule? (There may be multiple items for each line.)

***f* has to satisfy:**

***g* has to satisfy:**

Other conditions:

4. Exercises.

(a) $\lim_{x\to\infty} x^3 e^{-x}$

(b) $\lim_{x\to 0} \frac{3^x - 1}{2^x - 1}$

(c) $\lim_{x\to\infty} x \sin(1/x)$

(d) $\lim_{x\to\infty} \frac{\sin x}{x}$

(e) $\lim_{x\to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

(f) $\lim_{x\to\pi/2} \frac{\sec x}{1 + \tan x}$

(g) $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$. Hint: find the limit of the ln of the given function.

(h) A questions asks $\lim_{x\to\infty}\frac{x+\sin x}{x}$. Consider the following "solution."

$$\begin{aligned}\lim_{x\to\infty}\frac{x+\sin x}{x} &\quad (=\text{“}\frac{\infty}{\infty}\text{”, use L'Hôpital's rule})\\ &= \lim_{x\to\infty}\frac{1+\cos x}{1}\\ &= \lim_{x\to\infty} 1+\cos x.\end{aligned}$$

Since $\cos x$ oscillates when x goes to infinity, the limit $\lim_{x\to\infty}\frac{x+\sin x}{x}$ does not exist.

Why is this solution incorrect? (Hint: refer back to the hypotheses you listed in section 3.)