Math 1110: **Review** Oct 25 (Fri)

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Consider attending one of the review sessions by Quincy Loney:
10/28 MLT 228 4:00-6:00, 10/29 MLT 251 2:55-4:10, 10/29 MLT 251 4:30-5:45

1. State the chain rule (making sure to include the hypotheses).

   If g is differentiable at x and f is differentiable at $g(x)$ then

   $$f(g(x))' = f'(g(x))g'(x).$$

2. What is the definition of "f is differentiable at a point x"?

   f is differentiable at x if $\lim_{z\to x} \frac{f(z)-f(x)}{z-x}$ exists.

   Equivalently, f is differentiable at x if $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ exists.

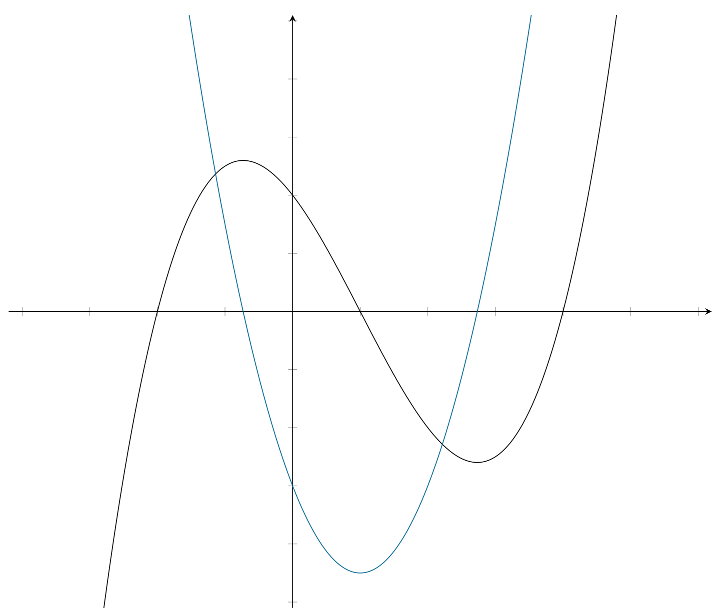
   (If x is an endpoint of the domain of f, then we only care about the one-sided limit.)

3. If f is differentiable at x, what is the geometrical interpretation of its derivative? How is that related to the secant lines of f?

   The derivative of f at x is the slope of the tangent line to the graph of f at x.

   The slope of that tangent line is the limit of the slopes of the secant lines of f between x and points near to x.

4. Sketch the derivative of the function below, on the same axes.



5. Find all points on the curve $y^4 = y^2 - x^2$ where the slope of the tangent line is zero. *The graph of the curve crosses itself at* $(0,0)$ *so there is no tangent line there.*

Differentiating both sides with respect to x, we get that

$$\begin{aligned} 4y^3y' &= 2yy' - 2x \\ 2yy' - 4y^3y' &= 2x \\ y'(2y - 4y^3) &= 2x \\ y' &= \frac{2x}{2y-4y^3} && \text{if } 2y - 4y^3 \neq 0 \\ &= \frac{x}{y(1-2y^2)} && \text{if } y(1-2y^2) \neq 0 \end{aligned}$$

If we solve for $y' = 0$, we get that

$$x = 0 \qquad \text{and} \qquad y(1-2y^2) \neq 0.$$

Substituting the equation on the left into $y^4 = y^2 - x^2$, we get that

$$y^4 = y^2 \qquad \text{and} \qquad y(1-2y^2) \neq 0.$$

The solutions to the equation on the left are $y = -1, 0, 1$. We are given a tangent line cannot exist at $(0,0)$.

So the points on the curve $y^4 = y^2 - x^2$ where the slope of the tangent line is zero are $(0,-1)$ and $(0,1)$.

6. Compute the derivatives of the following functions. On what domain is your answer valid?

(a) $5^{\sqrt{x}}$

$$(5^{\sqrt{x}})' = (\exp(\sqrt{x}\ln 5))' = \exp(\sqrt{x}\ln 5)\cdot\frac{\ln 5}{2\sqrt{x}} = \frac{5^{\sqrt{x}}\ln 5}{2\sqrt{x}}.$$

$5^{\sqrt{x}} = \exp(\sqrt{x}\ln 5)$ is differentiable on $(0,\infty)$, so the above is valid for $x \in (0,\infty)$.

(b) $\arccos(1/x)$

$$(\arccos(1/x))' = -\frac{1}{\sqrt{1-(1/x)^2}}\cdot -\frac{1}{x^2} = \frac{1}{x^2\sqrt{1-x^{-2}}} = \frac{1}{x\sqrt{x^2-1}}$$

arccos is only differentiable on $(-1,1)$, so the above is valid for x such that $1/x \in (-1,1)$, i.e., $x \in (-\infty,-1) \cup (1,\infty)$

7. Using linearization, compute an approximation for $\sin(3.1)$.

The linearization to $\sin(x)$ at the point a is $L_a(x) = \sin(a) + \cos(a)(x-a)$.

π is close to 3.1 and $\sin(\pi)$ and $\cos(\pi)$ are easy to compute, so we can use $L_\pi(3.1)$ to approximate $\sin(3.1)$:

$$L_\pi(3.1) = \sin(\pi) + \cos(\pi)(3.1-\pi) = 0 + (-1)(3.1-\pi) = \pi - 3.1 \approx 0.0416.$$