Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Jun Le Goh and Yuwen Wang

1. Objectives.

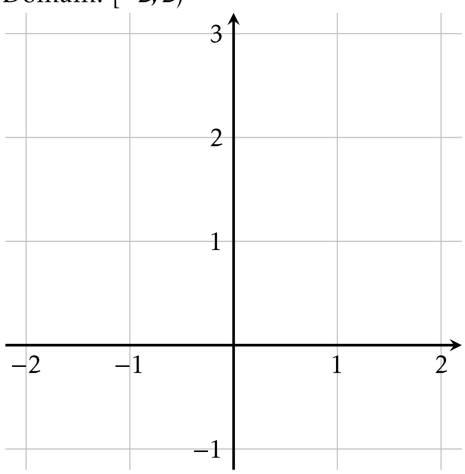
- define the notions of local/absolute min and max, and critical point,
- explain the extreme value theorem (in particular its hypotheses) and exhibit "counter-examples", i.e. functions that don't have an absolute min or max,
- find the absolute min and max of a continuous function on a closed interval [a, b].

2. Definition.

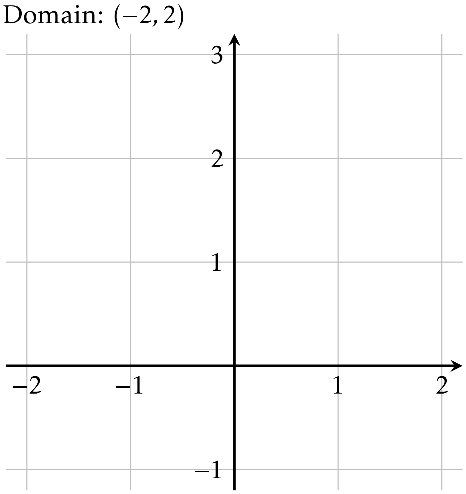
A function f has an *absolute maximum* (also known as a *global maximum*) at $x = c$ if $f(c)$ is the highest value of f anywhere; more precisely, f has an absolute maximum at $x = c$ if $f(c) \geq f(x)$ for all x in the domain of f. An absolute minimum is defined similarly.

If possible, create graphs of functions satisfying each description:

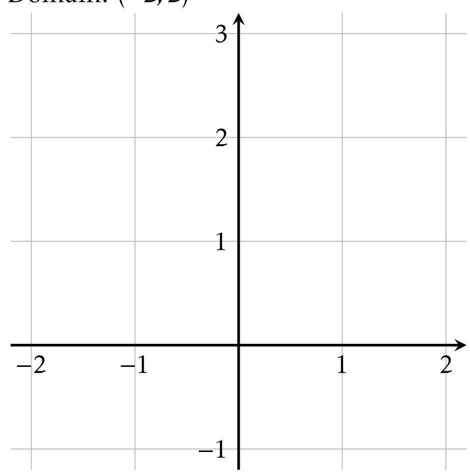
(a) A continuous function with an absolute maximum of 3 and no absolute minimum.
Domain: $[-2, 2)$



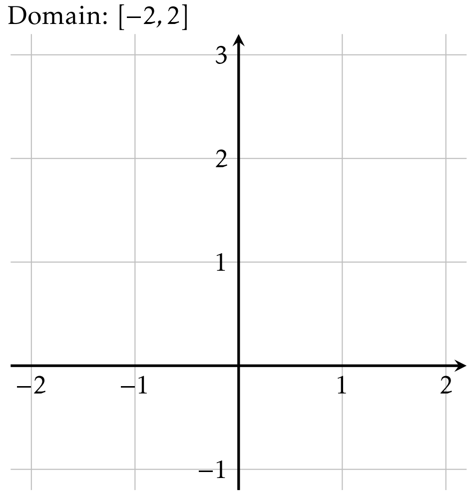
(b) A continuous function with an absolute maximum of 3 and an absolute minimum of -1.
Domain: $(-2, 2)$



(c) A continuous function with no absolute maximum and no absolute minimum.
Domain: $(-2, 2)$



(d) A continuous function with no absolute maximum and no absolute minimum.
Domain: $[-2, 2]$



3. EVT and its hypotheses.

The Extreme Value Theorem (EVT) states that:

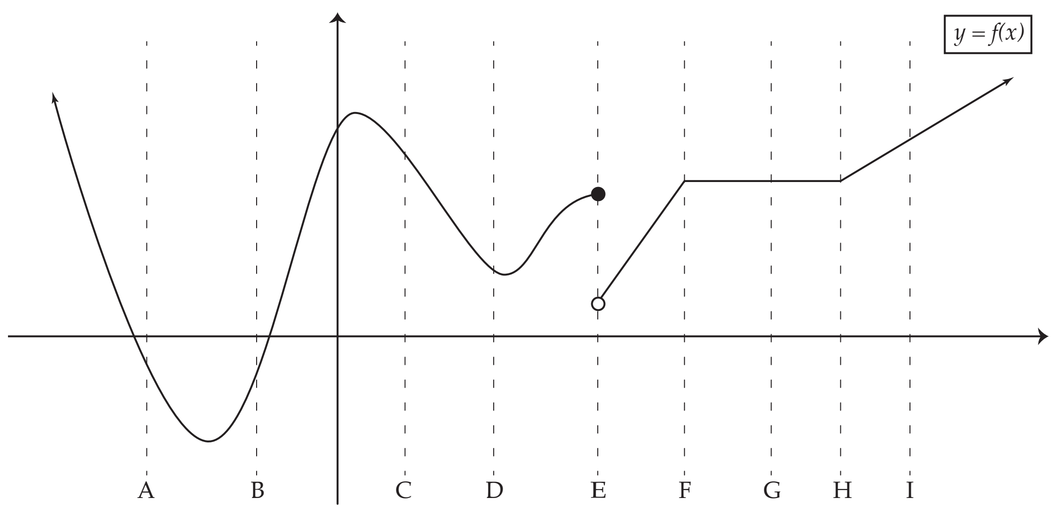
> If f is a continuous function with domain $[a,b]$, then f must have a global maximum value at some point in $[a,b]$. f must also have a global minimum value at some point in $[a,b]$. We say that f *attains* its global maximum/minimum in the interval $[a,b]$.

What are the hypotheses and the conclusions of EVT?

Hypotheses:

Conclusions:

A portion of the graph of the function $f(x)$ is shown in the figure below.



For each of the questions below, circle ALL of the available correct answers.

(a) On which intervals does $f(x)$ satisfy the hypotheses of the *Extreme Value Theorem*?

$[A,C]$ $[A,F]$ $[B,E]$ $[D,F]$ $(G,I]$ None

(b) On which intervals does $f(x)$ satisfy the conclusion of the *Extreme Value Theorem*?

$[A,C]$ $[A,F]$ $[B,E]$ $[D,F]$ $(G,I]$ None

On Friday, we will learn how to find global maxima and minima of functions by computing their derivatives. Based on the graph above, if f attains global max or min at the point a and f is differentiable at a, what can you say about $f'(a)$?