Math 1110: **Chain Rule** Sept 28 (Fri)

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1. Objectives.

- recognize when the chain rule is needed
- appropriately apply the chain rule to compute derivatives of functions

2. Chain rule.

Let f and g be functions. To apply the chain rule we need to check the following:

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Then we can conclude

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3. Chain rule application.

Compute the derivatives of the following functions (using the appropriate rules). Here you do **NOT** need to simplify your answer.

(a) $f(t) = \cos(t^2)$

(b) $g(x) = \sqrt{2x^3 + 4x + 2}$

(c) $h(z) = 2e^{z^2+4z} + 5z + 3$

(d) $k(x) = \sin(x) \cdot (x^2 + 5x)^{100}$

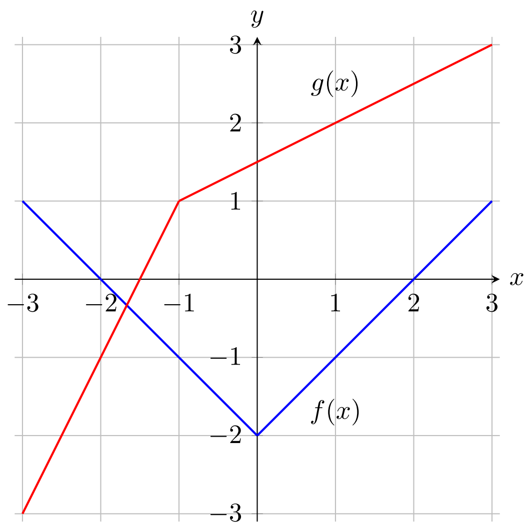
(e) $g(t) = \dfrac{e^t + 1}{\sin(t^4)}$

(f) $k(z) = e^{\cos(z^2)}$

(g) $f(x) = \tan^3\left(\sqrt{x^5 + 2}\right)$

4. Chain rule from graphs.

Consider the two functions $f(x)$ and $g(x)$ below.



(a) Let $h(x) = f(g(x))$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

(b) Let $h(x) = f(f(x))$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

(c) Let $h(x) = (f(x))^2$. Find: $h'(-2)$, $h'(0)$ and $h'(1)$.

5. Extra practice.

(a) Compute the derivatives of the following functions (using the appropriate rules). Here you do **NOT** need to simplify your answer.

(i) $f(x) = \sin^2(x) \cdot (x^2 + 5x)^{100}$

(ii) $h(t) = \dfrac{e^{t^3+t} + 1}{\sin\left(t^4\right)}$

(b) Use the chain rule to find the derivative of $\dfrac{1}{g(x)}$.

(c) Use part (b), the chain rule and the product rule to prove the quotient rule.