Math 1110: **Limits** Aug 31 (Fri)

Created by S. Bennoun, M. Hin, and T. Holm ©, modified Yuwen Wang.

1. Using the computations you have done in the pre-class activity as well as the Limit Laws (Theorem 1, p. 66, also on the next page), compute the following limits. For (b) and (c) justify each step of your computations.

   (a) $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 2}$,

   (b) $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$,

   (c) $\lim_{h \to 0} \frac{\sqrt{7h + 9} - 3}{h}$,

   (d) $\lim_{t \to -1} \frac{t^2 + 3t + 2}{t^2 - 1}$,

   (e) $\lim_{x \to -2} \frac{x + 2}{\sqrt{x^2 + 5} - 3}$,

THEOREM 1—Limit Laws

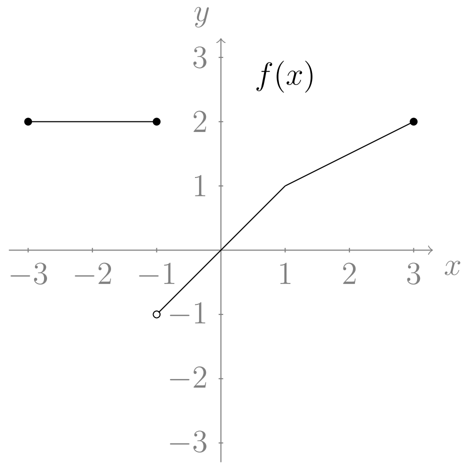
If L, M, c, and k are real numbers and

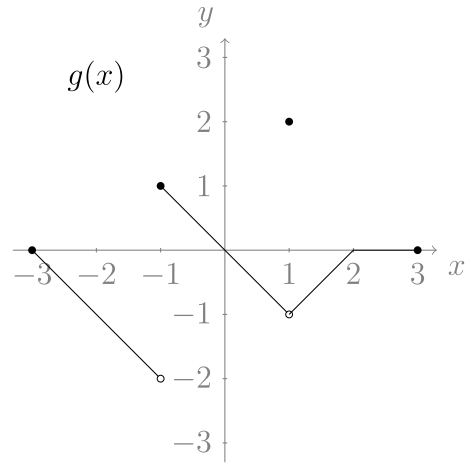
$$\lim_{x \to c} f(x) = L \quad \text{and} \quad \lim_{x \to c} g(x) = M, \quad \text{then}$$

1. *Sum Rule:* $\lim_{x \to c}(f(x) + g(x)) = L + M$
2. *Difference Rule:* $\lim_{x \to c}(f(x) - g(x)) = L - M$
3. *Constant Multiple Rule:* $\lim_{x \to c}(k \cdot f(x)) = k \cdot L$
4. *Product Rule:* $\lim_{x \to c}(f(x) \cdot g(x)) = L \cdot M$
5. *Quotient Rule:* $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \quad M \neq 0$
6. *Power Rule:* $\lim_{x \to c} [f(x)]^n = L^n$, n a positive integer
7. *Root Rule:* $\lim_{x \to c} \sqrt[n]{f(x)} = \sqrt[n]{L} = L^{1/n}$, n a positive integer

(If n is even, we assume that $f(x) \geq 0$ for x in an interval containing c.)

2. Here are the graphs of the functions f and g. Compute the limits indicated below.





(a) $\lim_{x \to 1} g(x)$

(b) $g(1)$

(c) $\lim_{x \to -1} f(x)$

(d) $\lim_{x \to -2} \frac{f(x)}{g(x)}$

(e) $\lim_{x \to 1} \frac{f(x)}{g(x)}$

(f) $\lim_{x \to 0} \frac{f(x)}{g(x)}$

(g) $\lim_{x \to 2} \frac{f(x)}{g(x)}$

(h) $\lim_{x \to -1} \frac{f(x)}{g(x)}$

3. Compute the following limits. If a limit does not exists because the right-hand and left-hand limits differ, evaluate them separately.

a) $\lim_{x \to 1} \frac{x^2 + 3x + 2}{(x-1)^2}$ b) $\lim_{x \to 1} \frac{x^2 + 3x + 2}{x - 1}$