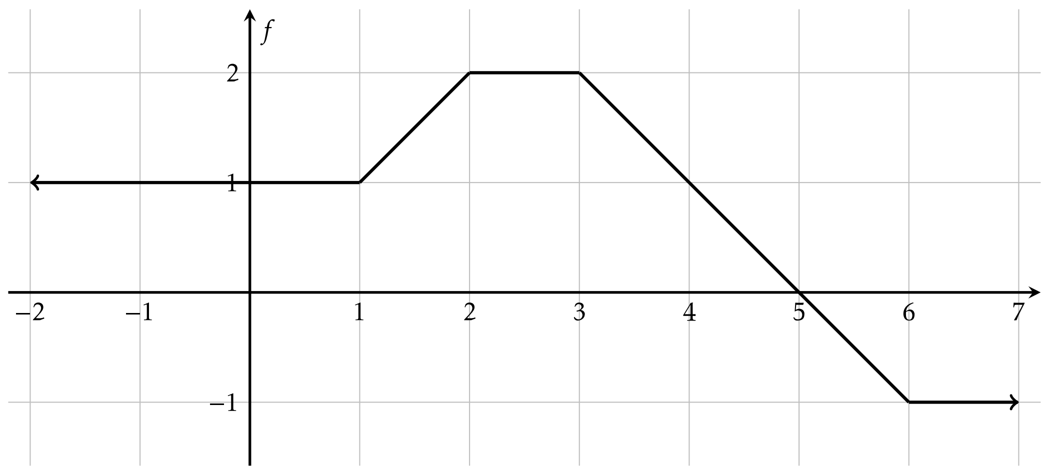
1. Area Accumulation Functions

Consider the function $f(x)$ whose graph is given below.



(a) Define $F(x) = \int_0^x f(t)\,dt$. What does this function represent?

(b) Fill in the chart below

x	-1	0	1	2	3	4	5	6	7
$F(x)$									

(c) Shade in and find the area represented by $F(3) - F(1)$.

(d) Find a formula for $F(x)$ between $x = 0$ and $x = 1$.

(e) Give the open intervals from -2 to 7 on which f increasing and decreasing. Explain.

2. Fundamental Theorem of Calculus (Part 1).

Let $f(t)$ be continuous function on $[a,b]$. Let $F(x) = \displaystyle\int_a^x f(t)dt$.

(a) Let $f(t) > 0$. Sketch the area $F(x+h) - F(x)$ where h is a small positive number.

(b) Approximate $F(x+h) - F(x)$ by the area of a rectangle.

(c) Use part 2 (b) to compute $F'(x) = \lim\limits_{h\to 0} \dfrac{F(x+h) - F(x)}{h}$.

3. Applying the Fundamental Theorem of Calculus.

Theorem. Let f be continuous on $[a,b]$ then $F(x) = \int_a^x f(t)\,dt$ is continous on $[a,b]$ and differentiable on (a,b). The derivative of F is

$$F'(x) = \frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

For the following questions, use the First Fundamental Theorem of Calculus to find $F'(x)$.

(a) $F(x) = \displaystyle\int_1^x \sqrt[4]{t}\,dt$

(b) $F(x) = \int_0^x \sec^2 t\, dt$

(c) $F(x) = \int_3^{5x^2+8} \cos(t)\, dt$ (Don't forget about the chain rule!)

(d) $F(x) = \int_{\sin(x)}^{x^2} \frac{1}{t^3}\, dt$ (Can you break this up somehow?)

4. Extra practice.

A particle is moving along the x-axis. The position of the particle at time t seconds is given by $x(t) = (t-1)(t-3)^2$, $0 \le t \le 5$. Find the total distance (in meters) the particle travels in 5 seconds.