1. Review of Riemann Sums.

Let f be a continuous function on $[a,b]$. A *Riemann sum for f on the interval $[a,b]$* is a sum of the form

$$S_P = \sum_{k=1}^{N} f(c_k)\Delta x_k,$$

where

- $P = \{x_0, x_1, \ldots, x_N\}$ is a *partition* of $[a,b]$, i.e.

$$a = x_0 < x_1 < \cdots < x_N = b,$$

- c_k is in $[x_{k-1}, x_k]$, and
- $\Delta x_k = x_k - x_{k-1}$.

Changing c_k we get different Riemann sums:

- For *left Riemann sum*, set $c_k = x_{k-1}$.
- For *right Riemann sum*, set $c_k = x_k$.
- For *midpoint Riemann sum*, set $c_k = (x_{k-1} + x_k)/2$.
- For *upper Riemann sum*, set $f(c_k)$ to be max f value in $[x_{k-1}, x_k]$.
- For *lower Riemann sum*, set $f(c_k)$ to be min f value in $[x_{k-1}, x_k]$.

2. Definition of definite integral.

Let f be a continuous function on $[a,b]$. The *definite integral of f from a to b* is the area under f from a to b, which can be evaluated as:

$$\int_a^b f(x)\,dx = \lim_{N\to\infty}\left(\sum_{k=1}^{N} f\left(a + k\frac{b-a}{N}\right)\frac{b-a}{N}\right),$$

as long as the limit exists.

Compute the following integrals by using area formulas for familiar geometric figures.

(a) $\displaystyle\int_0^1 x\,dx$

(b) $\int_{-2}^{2} 2 - |x|\,dx$

(c) $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$

(d) $\int_{-1}^{1} x\,dx$