Math 1110: **Concavity and Curve Sketching (4.4)** Oct 31 (Wed)

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1. Objectives.

At the end of this section, you will be able to:

- explain the difference between concave up and concave down,
- use the second derivative of a function to determine:
  - on what interval(s) a curve is concave up, respectively concave down,
  - where the inflection points are,
  - the nature of a local extremum,
- qualitatively sketch the graph of a function using the information provided by the first and second derivatives,
- given the algebraic expression of a function as well as its graph (e.g. using a graphing software), qualitatively verify that the curve corresponds to the given function.

2. Concavity.

Definition. A differentiable function $f(x)$ is
- *concave up* on an open interval I if f' is increasing on I.
- *concave down* on an open interval I if f' is decreasing on I.

A function that is concave up on its entire domain is *convex*.

Consider the function $f(x) = x^3 + 3x^2 - 1$. On which intervals is the function concave up, respectively concave down?

Second Derivative Test. A twice differentiable function $f(x)$ (on an interval I) is
- *concave up* on an open interval I if
- *concave down* on an open interval I if

3. Inflection points.

Definition. A point $(c, f(c))$ is a *point of inflection* if f has a tangent line at the point and the concavity changes.

Compute the inflection points for the following functions:

(a) $f(x) = x^3$

(b) $f(x) = \sqrt[3]{x}$

(c) $f(x) = x^4$

(d) $f(x) = x^3 + 2x^2 - 1$ (refer back to part 2.)

4. Curve sketching.

How to make a good curve sketch? Here are the main steps.

1.

2.

3.

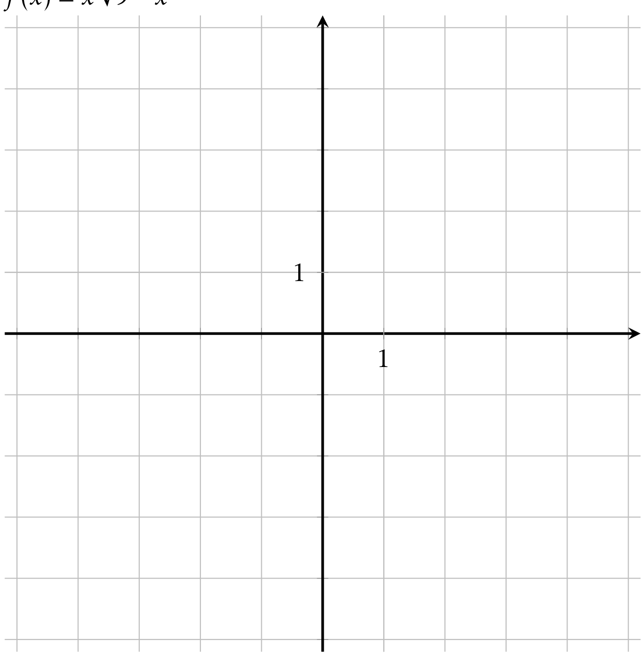
4.

5.

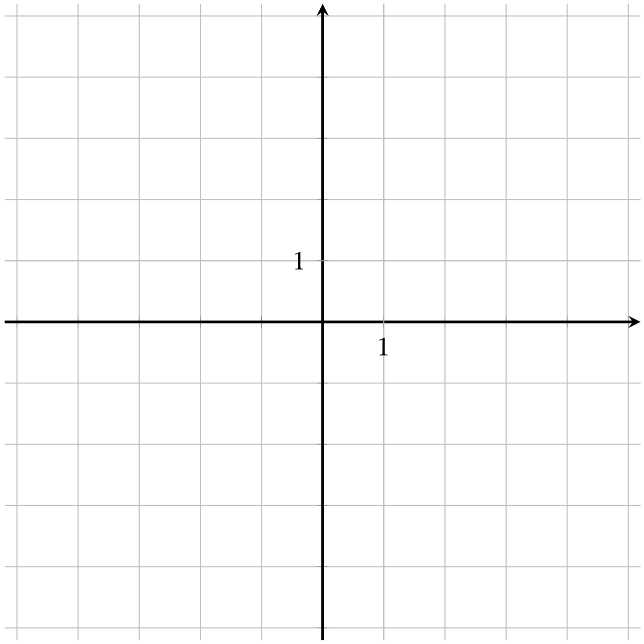
6.

Using the procedure described above, sketch the following functions:

(a) $f(x) = x\sqrt{9 - x^2}$



(b) $f(x) = \dfrac{x^2 - 3}{x^2 - 4}$



(c) $f(x) = \sqrt{|x|}$