Math 1110: Concavity and Curve Sketching

Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Yuwen Wang

In this activity we look at two increasing functions. We will see how we can describe the shape of their graphs more precisely by looking at the second derivative.

Consider the functions $f(x) = x^2$ and $g(x) = \sqrt{x}$ on the interval (0, 1).



- 1. Are these functions increasing or decreasing? Verify it by computing the derivatives f'(x) and g'(x).
- 2. Draw the tangent lines (by hand) for x = 0.25, 0.5, 1.5.In these two examples, where are the tangent lines situated with respect to the functions?
- 3. How do the slopes of the tangent lines change as *x* increases?

4. Compute the second derivatives f''(x) and g''(x). What are their signs on (0,1)?

5. What can we say about the relationship between the shape of a function and the sign of the second derivative?