Math 1110: **Mean Value Theorem** Oct 22 (Mon)

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Learning goals:

1. explain Rolle's Theorem and the Mean Value Theorem,
2. explain the importance of the Mean Value Theorem,
3. use the Mean Value Theorem to prove properties of a function based on information about its derivative.

Rolle's Theorem states that:

> Suppose that f is a continuous function on $[a,b]$, and that f is differentiable at every point in (a,b). If $f(a) = f(b)$, then there is some point c in (a,b) such that $f'(c) = 0$.

What are the hypotheses and the conclusions of Rolle's Theorem?

Hypotheses:

Conclusions:

Rolle's Theorem can be applied to analyze zeroes of functions, by looking at zeroes of their derivative. Use Rolle's Theorem to show that $x^3 + 3x + 1 = 0$ has at most one solution.
(Hint: What if it has two solutions a and b? Find some function f and then apply Rolle's.)

Rolle's Theorem is also a crucial ingredient in the proof of the Mean Value Theorem (MVT):

> Suppose that f is a continuous function on $[a,b]$, and that f is differentiable at every point in (a,b). Then there is some point c in (a,b) such that
> $$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

We can use MVT to easily prove Rolle's Theorem. How?

What are the hypotheses and the conclusions of the Mean Value Theorem (MVT)?

Hypotheses:

Conclusions:

In Australia and New Zealand, there are "point-to-point" speed cameras, which measure how long it takes a vehicle to travel from point A to point B. How does that help the traffic police determine whether vehicles are speeding?
Is there any scenario where a driver might be able to fool the speed camera into thinking that they are not speeding?

How does the MVT apply to this situation? (Find an appropriate function f, apply MVT to f, and describe what the conclusions of MVT mean in real life.)
What simplifying assumptions do we need to make?

If you have time, here's a question from HW9: Suppose that f is a continuous function on $[a,b]$, and that $f'(c) = 0$ for all $c \in [a,b]$. Show that f is constant on $[a,b]$.