Math 1110: **Critical Points and Global Extrema** Oct 19 (Fri)

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1. Using critical points to find .

We now know a continuous function defined on a closed interval will have an absolute (global) maximum and an absolute minimum. Often, our goal is to find the absolute minimum or maximum (if it exists) of a function on a given interval.

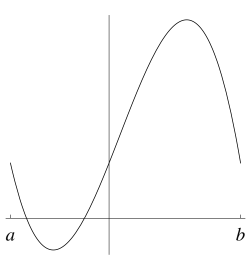
The new vocabulary words are:

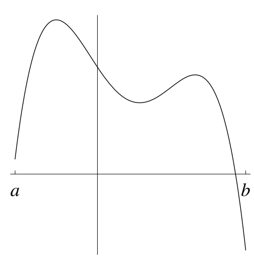
Local maximum/minimum:

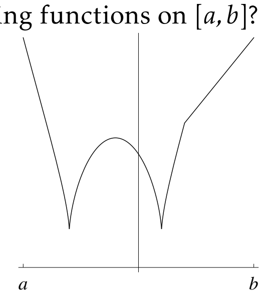
Relative extremum:

Critical points:

Using the vocabulary word(s), how would you find the global extrema of the following functions on $[a, b]$?







The steps for finding absolute extrema of a continuous function f on an interval $[a, b]$:

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3. Finding Extrema.

(a) Does $f(r) = 2\pi r^2 + \frac{256\pi}{r}$ have an absolute maximum and absolute minimum on $[1, 8]$? If so, where? (Give the values of r at which the absolute minimum and absolute maximum occur.)

(b) Does f have an absolute minimum and absolute maximum on $(0, \infty)$? If so, where?

4. A word problem.

A soda company wants its aluminium soft drink cans to have a volume of 128π cubic centimeters. In order to conserve resources, the company wants to minimize the amount of aluminium needed for each can. Let's use calculus to find the optimal dimensions of the can.

(a) What shape should we model the soda can as? Draw a picture.
(b) Assign variables to the relevant dimensions of the shape. Label your picture.
(c) Since we want to minimize the amount of aluminium needed for the can, which quantity, in terms of your variables, should we minimize?
(d) Eliminate any extra variables in the expression you just wrote.
(Hint: eliminate the height.)
(e) Where (if at all) does the above expression attain a global minimum?
(f) Suppose that the company wants the can to be at least 2 cm tall, with radius at least 1 cm. What are the optimal dimensions of the can?