Math 1110: **Linearization** Oct 15 (Mon)

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1. Objectives.

- explain in words what the process of linearization consist of and why it is interesting,
- use the linear approximation of a function at a given point to compute an approximate value of the function,
- using the graph of a function explain if a linear approximation gives an underestimate or overestimate of the true value of the function,
- explain in general terms what the conditions are for the process to give a "reasonable" approximation.

2. Linearization of the square root function.

Let's use linearization to approximate the value (the decimal value) of $\sqrt{10}$.

(a) What is a fundamental difference between $\sqrt{9}$ and $\sqrt{10}$?

(b) How could we use what you have done in the pre-class activity to approximate a function?

(c) Approximate $\sqrt{10}$. Keep some space on the right-hand side of the sheet.

3. Linearization of $1/x$.

Approximate 1/4.9 using an approximate linearization.
Use the steps you have identified in the previous part.

4. Linearization of e^x.

We want to compute an approximation of $e^{0.1}$ and e $(= e^1)$ using the tangent line. *Assume that we don't know the exact value of e.*

(a) Compute the appropriate linear approximation $L(x)$. What is the function?

(b) Using this approximation for $e^{0.1}$ and e, are we underestimating or overestimating the actual values of $e^{0.1}$ and e?

(c) If we compare the approximations for $e^{0.1}$ and e, which one is closer to the actual values of $e^{0.1}$ and e? What are your arguments to support your answer?

5. Extra practice: more on $\sqrt{x}$.

(a) In the pre-class activity, you looked the tangent lines to $\sqrt{x}$ at both $x = 1$ and $x = 9$. What difference have you noticed?

(b) Let us approximate $\sqrt{2}$ by using the linearization (i.e. the tangent line). What do you get?

(c) Using a calculator, compute the error of this approximation (i.e. the difference between this approximation and what you get with your calculator). Also compute the error of the approximation of $\sqrt{10}$ we did before.

(d) What do you notice? How can we explain this difference? What factors explain this difference?