Math 1110: **Related Rates** Oct 10 (Wed)

Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Yuwen Wang and Jun Le Goh

1. Objectives.

At the end of this section, students will be able to:

- build an appropriate mathematical model for word problems. This includes:
  - assign variables to appropriate quantities,
  - identify which numerical information is relevant and/or needed,
  - relate the variables using appropriate equations taking into account the numerical information provided,
- solve word problems using the differentiation techniques seen earlier in the term,
- for a given problem, clearly explain with words, mathematical symbols and equations their reasoning, in particular, what is known, what we are looking for and the steps of the procedure to solve the question.

2. Conical Tank.

When complete drainage is desired, conical tanks are commonly used. Consider a conical tank with top radius 4 ft and depth 10 ft. It is being filled with liquid at a rate of 2 ft^3/min. How fast is the liquid level rising when the liquid level is 5 ft?

(a) Draw a picture representing the situation.

(b) Assign variables to appropriate quantities. (To do so, it is helpful to consider what numerical information is provided.)

(c) Write an expression (in terms of your variables) for the desired quantity. In this case, the desired quantity is the rate of change of liquid level.

(d) Find an equation relating your variables.

(e) Differentiate to find an equation relating the desired quantity and other quantities (for which you are given numerical information).

(f) Use the above equation and the given values to find the desired quantity.

3. Spaaaaaaaaaaace!!

At the Kennedy Space Center, the VIP site for launch viewing (where a camera is located) is 4 mi away from pad 39A (where the shuttle is launched). The camera is set up to track the shuttle as it rises through the atmosphere. At 2 minutes after launch, the shuttle is 28 mi in the air, and it is traveling at 3000 mi/h. At that moment, how fast is the angle that the camera makes with the horizontal increasing?
Please present your solution properly (e.g., using a structure similar to that in the previous question.)

4. In-car speed camera.

An undercover police car equipped with an in-car speed camera is driving along a road toward a right-angled four-way intersection. A car is driving on the perpendicular road. The speed camera measures the speed at which the car is driving away from the police car. We want to determine the actual speed of the car.

(a) Draw a sketch of the situation. What assumptions are we making here?

(b) Assign variables to appropriate quantities and write expressions (in terms of your variables) for (1) the actual speed of the car; (2) the speed measured by the speed camera; (3) the speed of the police car. Do not differentiate anything yet.

(c) Find an equation that relates your variables.

(d) Differentiate to find an equation that relates (1), (2), and (3).

(e) To determine the speed of the car, what numerical information do we need?

(f) When the speed camera takes its measurement, the police car is 30 m away from the intersection and driving toward it at a speed of 20 km/h. The car is 40 m away from the intersection and driving away from it. The speed measured by the speed camera is 50 km/h. What is the speed of the car?

5. Blowing up a balloon.

You are preparing the birthday party of your 5-year old cousin. An essential element is of course to have balloons all around the room!
You use a pump (or your lungs), for which we know the output, to blow the balloons. We want to compute how fast the radius of a balloon increases as we blow it.

(a) Draw a sketch of the situation. What assumptions are we making here?
(b) Assign variables to appropriate quantities.
(c) Find an equation that relates your variables. Don't differentiate yet.
(d) Differentiate to find an equation.
(e) To determine how fast the radius of the balloon increasing, what numerical information do we need?
(f) If the pump you are using (or you lungs!) has an output of 50π cm^3/s, how fast is the radius increasing when the radius of the balloon is 5 cm.

6. Challenge Problem.

Connecting rods are used in internal combustion engines (i.e. most of today's car engines) or formerly in steam engines. Their role is to transform a linear movement into a circular one. For an illustration or this mechanism see https://upload.wikimedia.org/wikipedia/commons/0/01/Slider_Crank_animation.svg and https://en.wikipedia.org/wiki/Crankshaft#/media/File:Cshaft.gif.[1]
Our goal for this exercise is to determine how the linear and circular velocities are related.

(a) Draw a sketch of the situation. What is constant and what is variable?

(b) Find an equation that relates the angular velocity of the crankpin (the part that has a circular motion) to the velocity of the piston (the part of the crankshaft that moves on the horizontal axis).

You will probably need to use the law of sines $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$ and the law of cosines $c^2 = a^2 + b^2 - 2ab\cos\gamma$. [2]

(c) If the radius of the circle around which the crankpin is moving is 5cm and the length of the connecting rod 15cm, at what speed is the piston moving in *cm/s* if the crankpin is moving at 2000 round per minute?

[1] For more on this, look at https://en.wikipedia.org/wiki/Crankshaft or https://en.wikipedia.org/wiki/Internal_combustion_engine.

[2] More detail on Wikipedia: https:en.wikipedia.org/wiki/Law_of_sines and https://en.wikipedia.org/wiki/Law_of_cosines.