## Math 1110: Inverse Trig Functions

Created by Jun Le Goh ©, modified by Yuwen Wang

### 1. Review of Inverse Trignometric Function.

For each of the following functions, pick a domain where the function is domain. Sketch the function, and state the range for that domain.

(a)  $\arcsin(x)$ 

Domain: Range:

(b)  $\arccos(x)$ 

Domain:

Range:

(c)  $\arctan(x)$ 

Domain: Range:

#### 2. Derivative of arcsin.

Last time, we saw that the derivative of an inverse function is  $(f^{-1})'(x) =$ This formula assumes that

- (i)  $f^{-1}$  exists on an interval around x and is differentiable at x;
- (ii) f is differentiable at  $f^{-1}(x)$ ;
- (iii)  $f'(f^{-1}(x)) \neq 0.$

Let's compute the derivative of  $\arcsin x$ .

(a)  $\cos(\arcsin x)$  looks pretty gross, but it can be simplified. Use a trigonometric identity (or draw a right-angled triangle) to show that

$$\cos(\arcsin x) = \sqrt{1 - x^2}.$$

(b) Based on the graph of arcsin, on what interval is it differentiable?

(c) Compute the derivative of  $\arcsin x$ .

# 3. Derivative of arctan.

(a) Complete the following trigonometic identities:

$$\sin^2 x + \cos^2 x =$$
$$1 + \tan^2 x =$$
$$\arcsin x + \arccos x =$$

(b) Use a trigonometric identity to show that

$$\sec^2(\arctan x) = 1 + x^2.$$

(c) Based on the graph of arctan, on what interval is it differentiable?

(d) Compute the derivative of  $\arctan x$ .

## 4. Extra problems.

(a) Using a trigonometric identity and the derivative of  $\arcsin x$ , compute the derivative of  $\arccos x$ . (Once you have the right trigonometric identity, this is really easy.)

(b) Compute the derivative of  $\arcsin(1-t)$ . On what interval is this answer valid?