Math 1110: **Derivatives of Inverse Functions (Thomas 3.8)** Oct 3 (Wed)

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1. The goal of this first part is to compute the derivative of the inverse function $f^{-1}(x)$ of a function $f(x)$ (and this way prove the formula).

   (a) Write down the chain rule.

   (b) Let $f(x)$ be a function and $f^{-1}(x)$ its inverse.
   What is $f(f^{-1}(x))$ equal to?

   (c) Take the equation from the preceding point and differentiate both sides with respect to x.
   What do you get? Have you used the chain rule? If so, where?

   (d) Use what you have found to determine the derivative $(f^{-1}(x))'$.

   (e) What do we need to check before using the previous formula?

Let us now apply what we have just seen.

2. We want to compute the derivative of $f(x) = \ln x$. How can we use the preceding formula?

3. Let us now focus on the derivative of $f(x) = a^x$.
   How can we rewrite this function in terms of e^x and $\ln x$?
   Then what is $(a^x)'$?

4. Compute the derivatives of:

i) $f(x) = \ln(\sin x)$

ii) $g(x) = \dfrac{1}{\ln 3x}$

iii) $h(t) = 3^{t^2}$

iv) $f(z) = \log_5 e^z$

v) $g(t) = \ln(e^{3t} \sin^2 t)$

vi) $h(x) = \log_2(2^x e^2)$

vii) What is wrong with the following statement?
The derivative of $f(x) = \ln(\ln x)$ is $f'(x) = \frac{1}{x} \ln x + \ln x \frac{1}{x} = \frac{2 \ln x}{x}$.