Math 1110: **Chain Rule** Sept 28 (Fri)

Created by S. Bennoun, M. Hin, and T. Holm ©, modified by Yuwen Wang

1. For each of the following functions, explain which differentiation rule(s) apply.
   (You don't need to actually compute the derivatives).

   (a) $f(x) = 2x^4 - 3\cos x$,

   (b) $g(t) = 5te^t$,

   (c) $h(z) = \dfrac{z^2}{\sin z}$

2. Let us now look at the function $f(x) = \sin(x^2)$. What differentiation rule(s) apply here?
   What do you think is $f'(x)$? Check your answer by graphing $f(x)$ and $f'(x)$ on GeoGebra or Desmos.
   Was your answer correct?

3. Same question with for $f(t) = \sin(4t)$? (This time you have to look more closely at the graph).

4. What is going on in these examples? Why "simply" applying directly the rules we have learned so far doesn't work? What feature makes the whole thing fail?

In class we will use composition of functions. It is thus important to understand how it works.

If $f(x) = \sin x$ and $g(x) = 3x + 1$, then $f \circ g(x) = f(g(x))$ is given by $x \xrightarrow{g} 3x + 1 \xrightarrow{f} \sin(3x + 1)$, i.e. $f(g(x)) = \sin(3x + 1)$.
Conversely, given the function $h(x) = e^{\sqrt{x}}$, we can decompose it as $x \xrightarrow{g} \sqrt{x} \xrightarrow{f} e^{\sqrt{x}}$. Here we thus have $h(x) = f(g(x)) = e^{\sqrt{x}}$ with $f(x) = e^x$ and $g(x) = \sqrt{x}$.

Fill in the blanks with the appropriate functions:

if $f(x) = 1/x$,	$g(x) = 3x^2 + x + 4$,	then $f(g(x)) = \ldots\ldots\ldots,$
if $f(x) = \ldots\ldots\ldots,$	$g(x) = \sin x$,	then $f(g(x)) = \cos(\sin x))$,
if $f(x) = e^x$,	$g(x) = \ldots\ldots\ldots,$	then $f(g(x)) = e^{2x^3}$,
if $f(x) = \ldots\ldots\ldots,$	$g(x) = 2x^2 + 3$,	then $f(g(x)) = \sqrt{2x^2 + 3}$,
if $f(x) = \tan x$,	$g(x) = \ldots\ldots\ldots,$	then $f(g(x)) = \tan(x^3 + x + 6)$.