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The goal of this exercise is to see how we can determine the slope of the tangent line to a function at a given point.

For this exercise, we consider a function $f(x)$ and we are looking for the slope of its tangent line at the point $x_{0}=1$. To do so, we will look at secant lines that pass through the point $\left(x_{0}, f\left(x_{0}\right)\right)=(1, f(1))$ on the function.

1. Draw each of the lines below and estimate their slopes using the grid pattern of the graph.
(a) The line that passes through $(1, f(1))$ and $(4, f(4))$,
(b) The line that passes through $(1, f(1))$ and $(3, f(3))$,
(c) The line that passes through $(1, f(1))$ and $(2, f(2))$,
(d) The line that passes through $(1, f(1))$ and $(1.5, f(1.5))$,

2. Draw the tangent line to the function $f(x)$ at $x_{0}$.
3. Which of the secant lines you have drawn is the closest to the actual tangent line at $x_{0}$ ? How could you improve this process of approximating the tangent line further?
4. Let us now look at the secant line that passes through $(1, f(1))$ and $(4, f(4))$.
(a) What is its slope?
(b) Rewrite the formula of the slope using only $1, f(1), 4$, and $f(4))$.
(c) Rewrite the formula of the slope using only $x_{0}, f\left(x_{0}\right), 4$, and $\left.f(4)\right)$.
5. For the line that passes through $(1, f(1))$ and $(2, f(2))$, write the formula of the slope using only $x_{0}, f\left(x_{0}\right), 2$, and $\left.f(2)\right)$.
