Math 1110: **Continuity** Sept 7 (Fri)

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1. Continuity

Definition: Let c be a real number that is either an interior point or an endpoint of an interval in the domain of f. The function f is *continuous at c* if

$$\lim_{x \to c} f(x) = f(c).$$

The function f is *right-continuous at c* if

$$\lim_{x \to c^+} f(x) = f(c).$$

The function f is *left-continuous at c* if

$$\lim_{x \to c^-} f(x) = f(c).$$

The function f is *continuous on* $[a, b]$ if it is right continuous at

(a) To check whether $f(x)$ is continuous at c, I need to check these three things:

(b) Which of the following functions are continuous? Drawing their graphs can be helpful. (for $g(x)$, it may be helpful to factor the numerator).

$$f(x) = x + 1, \qquad g(x) = \begin{cases} \frac{x^2-1}{x-1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}, \qquad h(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases},$$

$$k(x) = \begin{cases} \frac{1}{x^2} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases} \qquad l(x) = \frac{1}{x}, \qquad m(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases},$$

(c) For functions with the discontinuities in the previous part, label the type of discontinuity: removable, jump, infinite discontinuities, and oscillating.

2. Intermediate Value Theorem

Theorem: If f is a continuous function on a closed interval $[a, b]$, and if y_0 is any value between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a, b]$.

Here is an application of the Intermediate Value Theorem (IVT).

(a) Does the equation $x^3 + 2x^2 - x = 1$ have a solution between $x = 0$ and $x = 1$.

(b) Show that the equation $-x^2 + 6x - 7 = 0$ has a solution between $x = 0$ and $x = 5$. How would you show that there are in fact two solutions?