Math 1110: Functions and Graphs

Aug 27 (Mon)

Created by S. Bennoun, M. Hin, and T. Holm ©, modified by JL Goh

1. Define $f(x) = \frac{x^2 - 4}{x - 2}$, g(x) = x + 2, and h(u) = u + 2. Which of these three functions are equal?

2. (a) Draw the graph of the function f(x) = |x| and write down the definition of this function (i.e. |x| = ...).

(b) How can we "modify" this function to shift it horizontally by 2 units to the right and vertically upward by 3 units (i.e. we want the vertex of the function to lie at (2,3))?

(c) Let $g(x) = x^2 - 3x - 4$. Are $f \circ g$ and $g \circ f$ equal?

3. Do we have $0 \cdot \infty = 0, 0 \cdot \infty = \infty$ or something else?

4. (a) Write down the domain and range of $f(x) = e^x$. On what domain is it increasing?

(b) Is $f(x) = e^x$ one-to-one? Is $f(x) = e^x$ invertible? If it is, write down $f^{-1}(x)$ as well as its domain and range.

(c) Using (a), find two different expressions which appear to be equal to x. When are these expressions well-defined?

- 5. In addition to the basic trigonometric functions of sine, cosine, and tangent, there are reciprocal functions as well. We will consider the secant function: $\sec x = \frac{1}{\cos x}$.
 - (a) Draw the graph of the function $f(x) = \sec x$ and write down the definition of this function.

(b) Let $g(x) = \arccos x$ be the inverse of the cosine function. Is f(x) = g(x)?